Quantification of Nonlocality in Quantum Information with Massively Parallel Computing

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1 INTRODUCTION

Entanglement is a key resource that is essential to make some of quantum information applications be advantageous against their classical counterparts. The motivation of entanglement quantification is thus obvious as it can be used to computationally explore the potential practicality of certain quantum circuits or states. The marginal operational quasiprobability (OQ) function is one of computational methods that can characterize the entanglement strength of quantum states by quantifying their nonlocality [2]. OQ is advantageous against the well-known full-state tomography method as it only involves directly measurable operators and generally requires a less number of measurements for verification. Its computing cost, however, is still nonnegligible and exponentially increases as the quantum-bit (qubit) size of target states grows, so the utilization of high performance computing resources must be pursued.

2 METHOD AND RESULTS

The focal numerical operations of the OQ function are the Kronecker product of Pauli matrices and the fast Fourier transform (FFT), where the first one is used to evaluate a set of expectation values for a quantum state with a predefined combinatory set of Pauli measurements. The corresponding probability distribution function (PDF) can be obtained by taking FFT of the set of expection values. The state nonlocality is then quantified by summing all the negative components of PDF and becomes 0 if the PDF is nonnegative. In terms of computing time, the major burden happens in the evaluation process of expectation values. This burden is parallelized in a 2-level manner with Message Passing interface (MPI) so a group of MPI processes conducts the combinatory computation in an embarrassingly parallel manner, and each MPI process here has a subgroup of MPI processes to handle Kronecker products.

The functionality of OQ is tested for verification of the entanglement swapping protocol [1] that is essential for realization of quantum repeaters. The 4-qubit computational target is shown in Figure 1(a), where the initial state is $|0000\rangle$. Here, two 1-qubit hadamard (H) operations followed by two 2-qubit controlled-X (CNOT) operations are conducted to generate two 2-qubit Bell-states (one between qubit 1 & 2 and the other between qubit 3 & 4). Sequential conduction of additional CNOT (qubit 2 & 3), H (qubit 2), and U (qubit 4, whose operation is determined by measured values C1 & C2 as shown in Figure 1(a))) entangles qubit 1 and 4, transforming two local entanglements (at Depth 2) into a single, long-distant one (at Depth 3). Figure 1(b) shows the nonlocality between two qubits that are calculated with OQ at Depth 1, 2, 3 of the circuit. At Depth 2, a positive value (0.125) is observed between qubit 1 & 2 and qubit 3 & 4 indicating the two local pairs are entangled. The successful conversion of these two entanglements into a single one can be verified with the result at Depth 3, where the same nonlocality is only observed between qubit 1 and 4. Figure 1(c) shows the parallel efficiency of OQ obtained in the NURION supercomputer [3] for verification of 4-6 qubit Greenberger-Horne-Zeilinger (GHZ) states.

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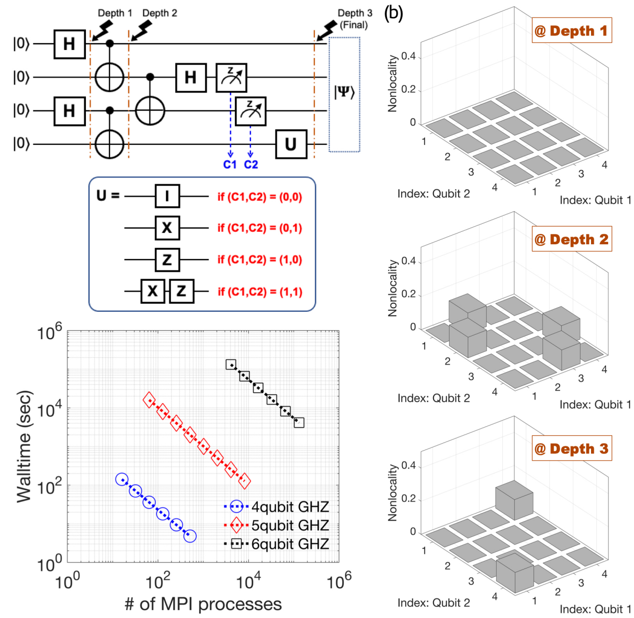

Figure 1: (a) A 4-qubit entanglement swapping circuit. (b) Results obtained at Depth 1-3 show the two local entanglements (Depth 2) are converted well into a long-distant one (Depth 3). (c) Walltime and strong scalability tested for verification of GHZ states in the NURION supercomputer.

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