Quantification of Nonlocality in Quantum Information with Massively Parallel Computing

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INTRODUCTION

Quantum Entanglement

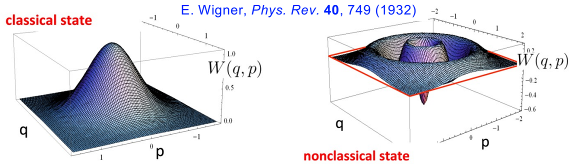
- One of key ingredients that is critical to drive advantages of quantum applications against their classical counterparts.
- **Entanglement quantification** paves the path for tackling a few interesting & important issues, e.g., (1) whether a given state is indeed entangled when measured with a finite set of physically realizable operators? (2) whether a circuit can be classically implemented?
- **Operational quasiprobability** function can serve as a cost-efficient indicator of entanglement with respect to a full-state tomography, but its computing expense still sharpy grows as the qubit-size of a target state increases.

METHODOLOGY

Operational Quasiprobability (OQ)

- **Negative probability** → a signature of nonclassicality

E. Wigner, *Phys. Rev.* **40**, 749 (1932)



[Wigner function (W) of a classical (left) and a nonclassical (right) state]

- In the case where K measurement operators are considered per qubit, the OQ function can be evaluated as follows:
  - Find 2^K expectation values ($\mathcal{C}$)
  - Take a discrete Fourier transform of expectation values ($\mathcal{W}$)
  - Expansion to a N-partite *system* is conceptually straightforward

$$\mathcal{C}(\mathbf{n}) = \sum_{\mathbf{a}} (-1)^{\mathbf{n}\cdot\mathbf{a}} P(\mathbf{a}|\mathbf{A_n})$$

[expectation value of a 1-qubt state]

$$\mathcal{W}(\mathbf{a}) \equiv \frac{1}{2^K} \sum_{\mathbf{n}} (-1)^{-\mathbf{a}\cdot\mathbf{n}} \mathcal{C}(\mathbf{n})$$

[OQ function of a 1-qubit state]

$$\mathcal{C}(\mathbf{n}^1, \ldots, \mathbf{n}^N) \equiv \mathcal{C}(\mathbf{n}^1) \otimes \cdots \otimes \mathcal{C}(\mathbf{n}^N)$$

$$\mathcal{W}(\mathbf{a}^1, \ldots, \mathbf{a}^N) \equiv \frac{1}{2^{NK}} \sum_{\mathbf{n}^1,\ldots,\mathbf{n}^N} (-1)^{-\mathbf{a}^1\cdot\mathbf{n}^1 \cdots - \mathbf{a}^N\cdot\mathbf{n}^N} \times \mathcal{C}(\mathbf{n}^1, \ldots, \mathbf{n}^N)$$

[Expansion to a N-partite system]

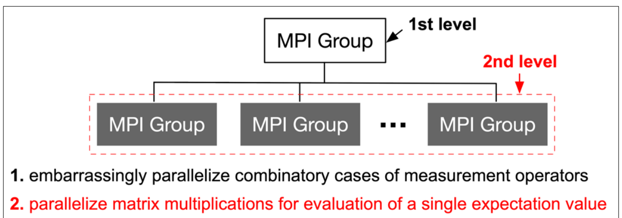
Computing Cost & Parallelization

- The hotspot is **the evaluation of expectation values** ($\mathcal{C}$).
  - In the case of N-partite systems, evaluation of a conditional probability involves $2K+1$ multiplications of a 2^Nx2^N matrix.

$$P(a_i, a_j | A_i, A_j) = \mathrm{Tr}[\Pi_j^{a_j} \Pi_i^{a_i} \varrho\, (\Pi_j^{a_j} \Pi_i^{a_i})^\dagger]$$

[Numerical expression: conditional probability when K = 2, N = 2]

  - Conditional probability is evaluated $2^{(N \times K)}$ times to fully fill the $\mathcal{C}$ vector, i.e., vector size: $2^{(N \times K)}$.
- Discrete Fourier transform is conducted with a FFTW library.
- **A two-level MPI parallelization scheme** is employed.



[A two-level MPI parallelization scheme for OQ calculations]

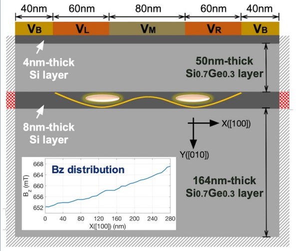
RESULTS & DISCUSSION

Entanglement in Semiconductor Devices

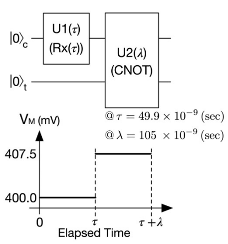
- **Entanglement characteristics in Si quantum dot systems** is studied with OQ coupled to device simulations: Entanglement strength is much more robust to device-inherent charge noise than state-fidelity.

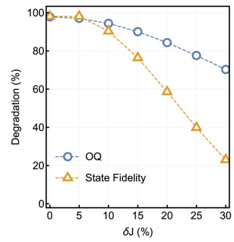
H. Ryu et al., *Scientific Reports* **12**, 15300 (2022)

J. Ryu et al., *Quantum* **6**, 827 (2022)



[Simulation domain: Si DQD systems]

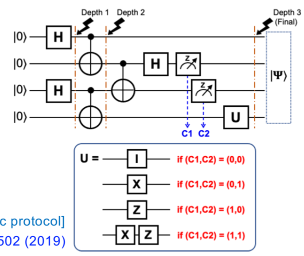




[Noise-sensitivity of a DQD-generated Bell-state]

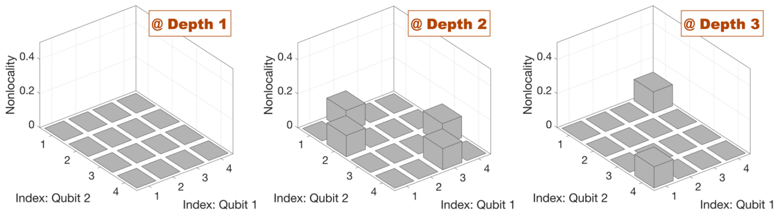
Entanglement Swapping (ESWAP) Circuit

- A 4-qubit ESWAP logic protocol is used as a target model problem to explore a step-by-step change in quantumness.
- OQ-driven calculations present a practical numerical way to conceptually understand the time-varying shift of nonlocality of a 4-qubit state, supporting the operation as an ESWAP protocol



[A 4-qubit ESWAP logic protocol]

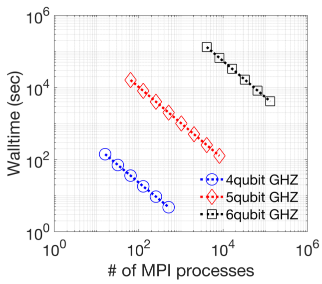
W. Ning et al., *Phys. Rev. Lett.* **123**, 060502 (2019)



[OQ result at each step of the 4-qubit ESWAP circuit]

Strong Scalability: MPI Parallel Efficiency

- Entanglement characteristic of N-qubit GHZ states (N = 4, 5, 6): K = 3 (three measurement operators (px, py, pz) are considered per qubit)
- OQ calculations are conducted in the NURION supercomputer (Intel® Xeon Phi KNL 7250 - 68 cores, 96GB DDR4, 16GB MCDRAM per node): Fairly nice scalability is secured in upto 2,048 KNL nodes (139,264 computing cores)



[Strong scalability of OQ calculations]

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